AUTOMATIC INVERTED PENDULUM CONTROL SYSTEM

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ABSTRACT

This project is about developing of an Inverted Pendulum that been control by PID controller. The Inverted Pendulum is a classical control system problem due its nonlinear characteristic and unstable behavior. It is have same concept like the hand (broom) to maintain the stick upright. An inverted pendulum is a pendulum attach directly with the sensor (potentiometer) at pivot point. The cart mounted directly on the tire to move horizontal either forward or backward. The movement of mechanical part can be determining using PID controller and implement to microcontroller. Microcontroller used in this project is Arduino UNO. PID controller is obtained using MATLAB through Simulink. Lastly, the performances of PID controller are test on hardware to maintain the pendulum upright. Based on the result, the inverted pendulum manages to reach stability.

ABSTRAK

Projek ini tentang pemantul songsang yang dikawal menggunakan kawalan PID. Pemantul songsang adalah masalah kawalan sistem klasik disebabkan ciri-ciri dan keadan yang tidak stabil. Konsepnya adalah sama seperti tangan yang mengimbangi batang untuk kekal stabil keatas. Pemantul songsang adalah pemantul yang berhubung secara terus kepada pengesan yang digunakan iaitu potentiometer. Kereta melekat terus dengan tayar untuk bergerak melintang sama ada kedepan atau kebelakang. Pergerakan bahagian mekanikal dapat ditentukan menggunakan kawalan PID dan mengaplikasikan kepada kawalan mikro. Kawalan mikro yang digunakan dalam projek ini adalah Arduino UNO. Kawalan PID diperolehi daripada MATLAB melalui simulasi. Akhirnya, tindakbalas kawalan PID diuji kepada perkakas untuk mengetahui mengimbang pemantul keatas. Keputusan menunjukkan pemantul songsang berjaya mencapai kestabilan.

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CHAPTER 1

INTRODUCTION

1.1 Background

The inverted pendulum is the classical control system problem. It has some concept like a hand as a cart and stick as a pendulum which is hand try balance the stick. In addition, the inverted pendulum have limited motion that only can move right and left meanwhile the hand which try to balance the stick has advantage can moving upward and downward. An inverted pendulum does basically the same thing. Just like the broom-stick, an inverted pendulum is an inherently unstable system. Force must be properly applied to keep the system intact. To achieve this, proper control theory is required.

The inverted pendulum is essential in the evaluating and comparing of various control theories. The inverted pendulum (IP) is among the most difficult systems to control in the field of control engineering. Due to its importance in the field of control engineering, it has been choose for final year project to analyze its model and propose a linear compensator according to the PID control law.

In new era, the concept of inverted pendulum is become important in daily life especially in field control system. The concept stability that show in inverted pendulum can be apply in real life application for example the helicopter already use concept stability to reject windup disturbance and also the missile that moving faster without have problem due to the concept stability.

1.2 Project Synopsis

Broadly speaking, the inverted pendulum is the classical control system. Due to unstable system (non-linear) this project interesting to explore and do some research. This project involves two main parts which is hardware and software. First of all, the hardware includes the mechanical part, driver motor circuit and microcontroller. On the others hand, the PID controller is the part of controller that used to control the movement of pendulum cart during disturbances. The calibration between hardware and software will make this project challenging for undergraduate student.

1.3 Objective

- To develop a mathematical modeling of PID control system.
- To design and build cart automatic inverted pendulum control system using PID controller.
- To analysis the stability of pendulum during movement of the cart.

1.4 Problem Statement

The problem of this project is to maintain the pendulum upright when the cart is moving forward and backward or when the disturbance is approach to the pendulum. A PID controller should be design to provide good control around the unstable equilibrium (180 degree). There are two general control objectives which is quick rejection of low frequency disturbances and quick tracking of step reference inputs.

1.5 Project Scopes

This project involves the developing of hardware and software. Hardware have a few part need to design and the fabricate according the actual plan. The hardware use is mechanical part made myself using the steel plate. Others equipment is a potentiometer and DC motor. Meanwhile, the software use is MATLAB to simulate and analyze the behavior of pendulum or cart to maintain the pendulum and calculate the maximum angle for the cart during disturbances force. A PID controller use as the whole program for the reaction of the cart during disturbances and control the dc motor to move the cart forward and backward. A potentiometer functions as a sensor to detect inclination of angle on pendulum to maintain upright.

1.6 Project Organization

In general, this report consist of 5 chapters where chapter one is introduction of this report. **Chapter 1** explain briefly about the inverted pendulum system which are include introduction, project objective, problem statement, project scope and lastly the project organization.

Chapter 2 is discussing about the literature review which explains the details of the research on the inverted pendulum system background. In this chapter, theory and concept is explained briefly to relate the project in real life.

Chapter 3 is methodology which describes about the method that being used to solve the problems. The methods used in this topic are flowchart, process and analysis data, model and data collecting.

On the other hands, **Chapter 4** will discuss about the experimental and the result obtained from the project. The result included the project limitation, result analysis and suggestion for the project enhancement.

Lastly, **chapter 5** will discuss about the discussion and conclusion of this project. This chapter will state some recommendation about model, circuit and controller.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

In this chapter will discuss about the research findings of literature reviews from past researches. This chapter discussing about the modeling of the inverted pendulum and others mechanism that used to design control system for inverted pendulum. A PID controller problem is used widely nowadays rather than others type of controller, especially neural network controller. In addition, PID controller much more practical and allows for much better adjustments to be made in the system. This literature review will also evaluate the sources and some information will be used as references to build real inverted pendulum.

2.2 Definition of Control

Control is defined the power to make decisions about something or decide what should happen.

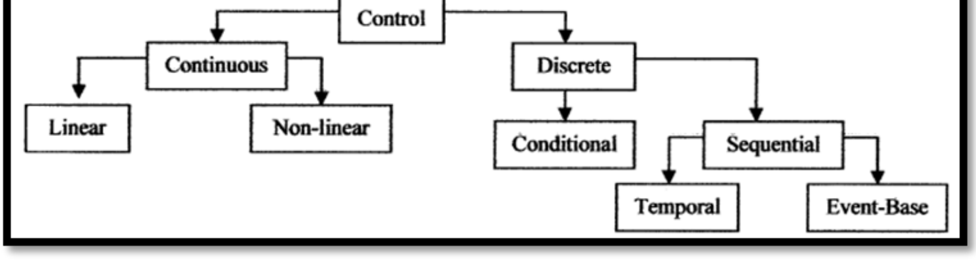

Figure 2.1 Types of control [10]

2.2.1 Control System

In most systems there will be an input and an output. A control system consists of subsystem and processes (or plant) assembled for the purpose of controlling the output of the processes. In the simplest form, a control system provides an output or response for a given input or stimulus .For example, the automobile steering which desired course of travel is known as input and actual course of travel known as output. In this system the others subsystem are driver, steering mechanism, and automobile. The Figure 2.2 shows the example of block diagram for control system.

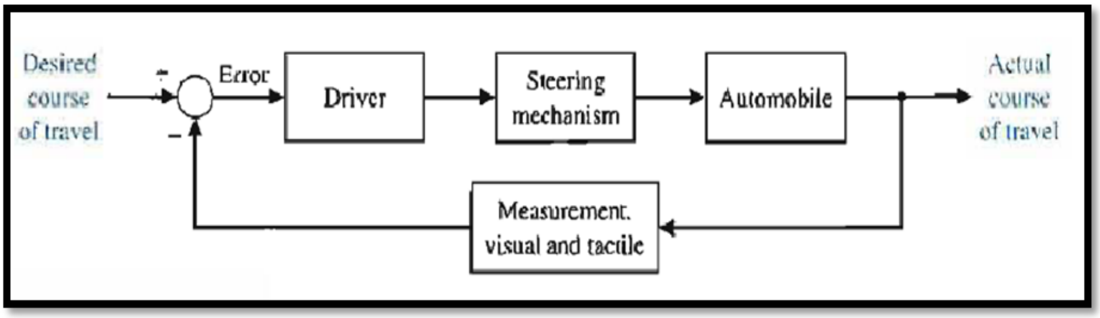

Figure 2.2 Block diagram automobile steering control system [10]

2.2.2 Open–Loop Control Systems

Open-Loop Systems start with a subsystem called an input transducer which convert the form of the input to the used by controller. The controller drives a process of plant. The input is sometimes called the reference, while the output can be called the controlled variable. Others signal such as disturbances, are shown added to controller and process outputs with summing junctions, which yield the algebraic sum of their inputs signal using associated sign.

Open–Loop System is the system does not use feedback. That’s mean the controller must independently determine the signal to send the actuator. The disadvantage with this approach is the controller never knows either the actuator do the right thing according the plan. (Feedback control principles)

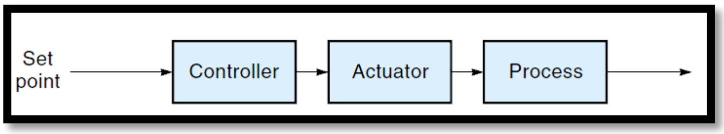

Figure 2.3 Open-loop control systems [1]

2.2.3 Closed-Loop Control Systems

Closed- loop system also known as a feedback control system. On the other hands, the output of process is constantly monitored by a sensor Figure 2.4. The sensor samples the output and passes this information back to controller due to the controller knows what actually system doing. In addition, the controller can make any adjustment necessary to keep the output where it belongs. This self-correcting feature of closed-loop control makes it preferable over open-loop control in application nowadays.(Feedback control principles)

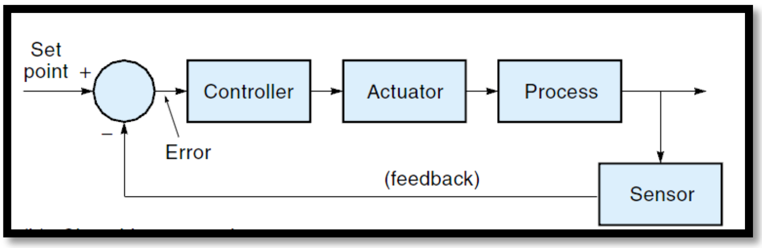

Figure 2.4 Closed-loop control systems [1]

2.3 Concept Model

Inverted pendulums have variety of model design nowadays. The pendulum cart concept will be discussed in aspect of modeling and dynamics. This pendulum cart inverted pendulum system same as seesaw system.

2.3.1 Pendulum Cart System

(Najihah, 2008) has presented the switching control for inverted pendulum system based on Energy Modification. To maintain the pendulum upright after disturbances the potential energy should be minimize. Broadly speaking, the approach to control the inverted pendulum system is accumulated by paying attention to the energy of pendulum. The model design is proposed where there is a mass situated at the centre of inverted pendulum. The equation derived using lagrangian dynamic analysis.

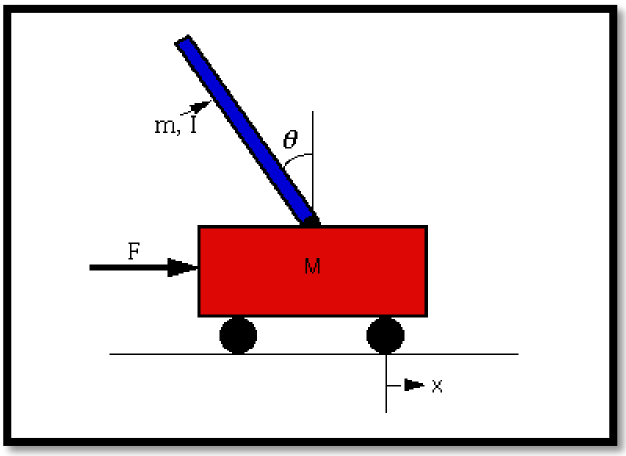

Figure 2.5 Model of pendulum cart system [2]

Table 2.1 Explanation the symbols

Quantity	**Symbol**
Mass of the cart	M
Mass of the pendulum	m
Friction of the cart	b
Length of the pendulum	l
Inertia of pendulum	I
Force applied to the cart	F
Cart position coordinate	x
Pendulum angle from vertical	θ

The Figure 2.6 below show the two free of body diagram (Hasan, Saha, Rahman, Sarker & Aditya, 2011):

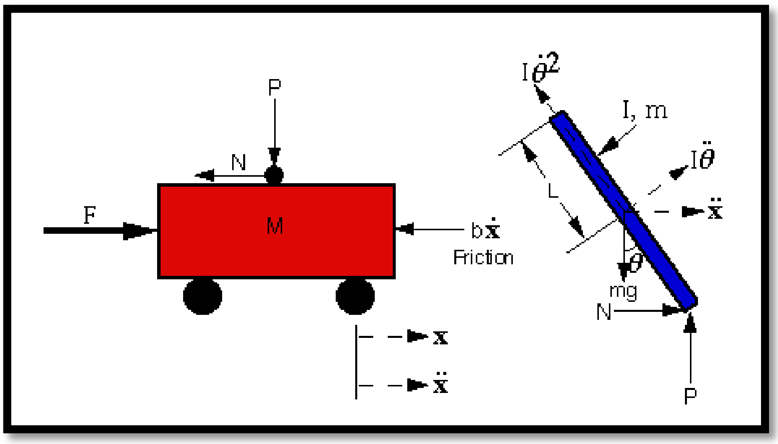

Figure 2.6 Free body diagram for cart and pendulum [3]

This system is difficult to design in Simulink because of the physical constraint (the pin joint) between the cart and pendulum which reduces the degrees of freedom in the system. Both the cart and the pendulum have one degree of freedom (X and θ, respectively). Then model Newton's equation for these two degrees of freedom.

$$\frac{d^2t}{dx^2} = \frac{l}{M}\sum_{cart} F_X = \frac{l}{M}\left(F - N - b\ \frac{dx}{dt}\right) \tag{2.1}$$

$$\frac{d^2\theta}{dx^2} = \frac{l}{I}\sum_{pent} \tau = \frac{l}{I}(NL\ \cos\theta\ + PL\ \sin\theta) \tag{2.2}$$

It is necessary, however, to include the interaction forces N and P between the cart and the pendulum in order to model the dynamics.

2.4 Mechanism

There are three main motor that can be used for this pendulum cart system. Firstly, a DC motor, second a stepper motor and lastly a servo motor. High torque and high speed is the main consideration for choosing the motor to fabricate in pendulum cart system. The torque is needed to change the direction and stabilize the pendulum in vertical. Meanwhile, the high speed is necessary so that the movement of cart can react faster to maintain pendulum.

(Lund, Aaen, Juliusson, Thorbergsson, Madsen & Kristensen, 2010) has presented the dc motor used is to develop torque on the pendulum. (Stang , 2005) state the DC motor could have high torque and high speed but it comes at a cost. In addition, the DC motor required more power to run when the speed and torque increase. DC motors also are operated closed loop that give faster response for feedback control. H-bridge control circuitry is used to control the direction of the DC motor either clockwise or vice versa based on the directional signal.

A stepper motor is electromechanical device which converts electrical pulses into discrete mechanical movements. On the other hand, the stepper motor have high torque but it problem with the speed in other words lack sufficient speed and resonances can occur if not properly controlled. It is also normally operated open loop and a stepper motor has two types of basic winding arrangement which is uni-polar and bi-polar.

In addition, a servo motor is the last choice to drive the cart but it is problem with the torque although could supply high speed. For the information, the servo motor has the ability to turn 360 degree only. This is problem for the cart to maintain the pendulum upright because over disturbances not cause by force but motor itself. Other problem is the voltage level applied to the motor tells the motor to be where and this is difficult to control it. In terms of controlling this part of motor less consideration to use for pendulum cart.

2.5 Sensor Feedback

There are 3 types of sensor common used in variety condition which is used in control system. Firstly are tilt sensor, second gyro sensor and lastly potentiometer. Three of them have their characteristic and different benefit. All types of sensor will be comparing to show their advantages and disadvantages.

Tilt sensor is the simple way to detect orientation or inclination. It is usually use in toys, gadgets and appliances. This sensor is small, inexpensive, low power and easy to use. (Hovingh & Roon , 2007) state the function of tilt sensor is to measure sideway acceleration of the pendulum due to gravity but the problem is this sensor can only detect acceleration and not position or velocity.

Tilt sensor also can calibration with Arduino. The Figure 2.7 below shows the simplest circuit connection with Arduino or microcontroller.

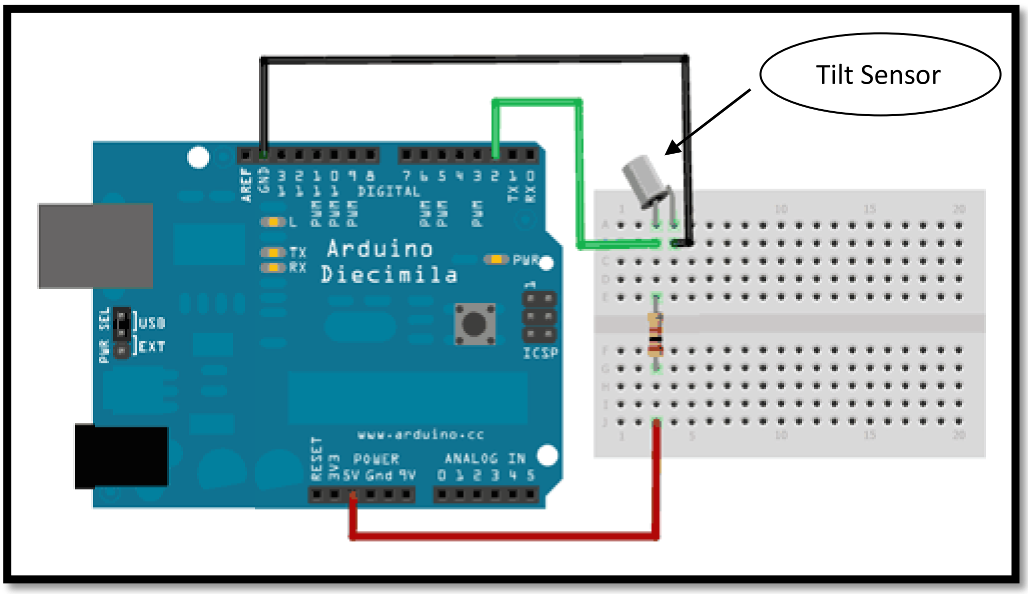

Figure 2.7 A simple circuit connection of tilt sensor with Arduino [9]

A Gyro sensor contains a single axis gyroscopic sensor that detects rotation and returns a value representing the number of degrees per second of rotation. In addition, this sensor can measure up $\pm$ 360 degree per rotation. A rotation rate can be read up to approximately 300 times per second is the other benefit of this sensor. According (Dexter, 1999-2004) the disadvantages of this sensor is expensive and noise when function.

Potentiometer is voltage regulator that can be used as sensor. (Hovingh & Roon , 2007) state the potentiometer is the suitable sensor that can be used due to inexpensiveness and simplistic nature. This sensor is sticking with the single axis for the forcing function maintains linearity for controlling the system. ("Position sensor,") state potentiometers come in a wide range of designs and sizes such as the commonly available round rotational type or the longer and flat linear slider types. When used as a positional sensor the moveable object is connected directly to the shaft or slider of the potentiometer and a DC reference voltage is applied across the two outer fixed connections forming the resistive element.

2.6 Controller Design

PID is the combination of Proportional, Integral and Derivative. This combination can be used to design the control system. In new era, more intelligence controller prefer designed that use op-amps or a microprocessor. Proportional control is the first and most basic that can be preferred.(Feedback control principles) said with proportional control, the actuator applies a corrective force that is proportional to the amount of error, as expressed in Equation 2.3:

$$Output_P = K_P E \tag{2.3}$$

In addition, the other controller is integral control. Integral control can reduce the steady-state error to zero in control system. The sum of all past errors multiplied by time is proportional to the integral control that creates restoring force, as expressed in Equation 2.4:

$$Output_t = K_I K_P \sum (E\Delta t) \tag{2.4}$$

The value of $\sum(E\Delta t)$ will increase with time, causing the restoring force to get larger in result for a constant value of error. Finally, the restoring force will get large enough to overcome friction and move the controlled variable in a direction to eliminate the error.

An analogy showing the power of integral control is a person who sits down in a comfortable chair to read a book. After a short time, the reader notices the dripping sound of a leaky faucet (steady-state error). The first response of the reader is to do nothing, but as time goes on the sink starts to fill up and spill over, which gets the reader's attention and he or she gets up and turns it off. The point is that the dripping (error) was not increasing, but the effect of the steady-state error was increasing with time until finally the reader (system) was motivated to do something about it (Feedback control principles).

The others control PID is derivative control that only solution to overshoot problem in control system. Derivative control functional "*applies the brake*" to slow down the controlled variable before it reaches destination. The equation below shows the contribution from derivative control in mathematical:

$$Output_D = K_D K_P \frac{\Delta E}{\Delta t} \quad (2.5)$$

On the other hand, the three combination control which is proportional + integral + derivative will be discussed on details. This combination as known as PID control and the foundation of the system is proportional control. Adding integral control may increase overshoot but a means to eliminate steady-state error. Meanwhile, derivative control is good to reduce the tendency to overshoot and also getting sluggish systems moving faster. Mathematically the combination of the PID control expressed in the Equation 2.6 below,

$$Output_{PID} = K_P E + K_I K_P \sum (E\Delta t) + K_D K_P \frac{\Delta E}{\Delta t} \quad (2.6)$$

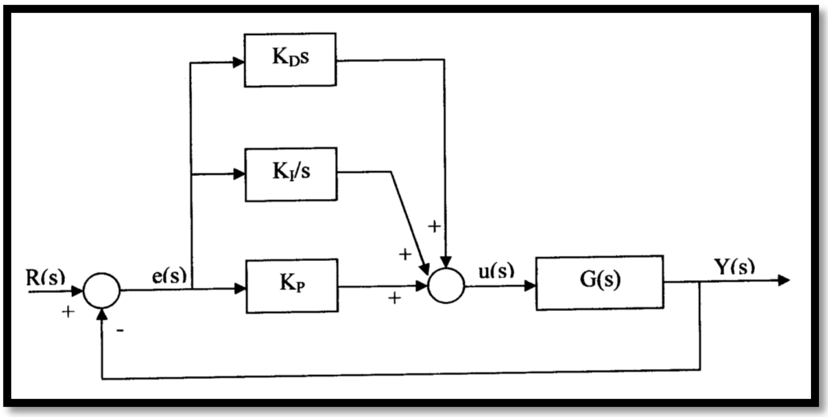

Figure 2.8 A PID controller system [1]

The comparison between proportional, integral and derivative summarize in the Table 2.2, in terms rise time, overshoot, settling time, and steady-state error.

Table 2.2 The comparison between K_P, K_I and K_D

Closed Loop Response	Rise Time	Overshoot	Settling Time	Steady-State Error
K_P	Decrease	Increase	Small change	Decrease
K_I	Decrease	Decrease	Increase	Eliminate
K_D	Small change	Decrease	Decrease	Small change

2.6.1 Analog PID

PID Equation 2.6 can be implementation either analog or digital. Unfortunately, all new installation used nowadays is digital control. (Aggarwal, Mao & O'Reilly) said the model using analog PID for self-tuning have several advantages over the digital PID system. Firstly, an analog system provides larger bandwidth, higher speed, and eliminates quantization noise. An analog controller must use a reconfigurable analog array instead of digital signal processor (DSP). The power consumption of a DSP halves every 18 months, as postulated by Gene's Law. Using a reconfigurable analog array can decrease power consumed by five orders of magnitude as compared to a DSP, implying by Gene's Law a 20 year leap in power reduction.

The next one, the analog controller save power, space and cost in addition it's also eliminates the need for ADCs and DACs. Figure 2.9 is the example of analog PID controller that uses five differential amplifiers in straightforward version. (Feedback control principles) state the first differential amplifier (U_1) subtracts the feedback from the set point to produce the error signal. Op-ampsU_2,U_3, and U_4 are known as unit gain, integrator and differential amplifiers, respectively. Other side, (U_5) sum all three terms which are U_2, U_3, U_4 and multiplies the sum by K_P to get output. The capacitor C_1 function to collect the error in form of charge meanwhile the capacitor C_D of the differentiator passes only the change in error. Besides, the constant K_P, K_I, and K_D are selected by adjusting R_1, R_2 and R_3. The circuits in Figure 2.9 can exactly implementation the PID equation.

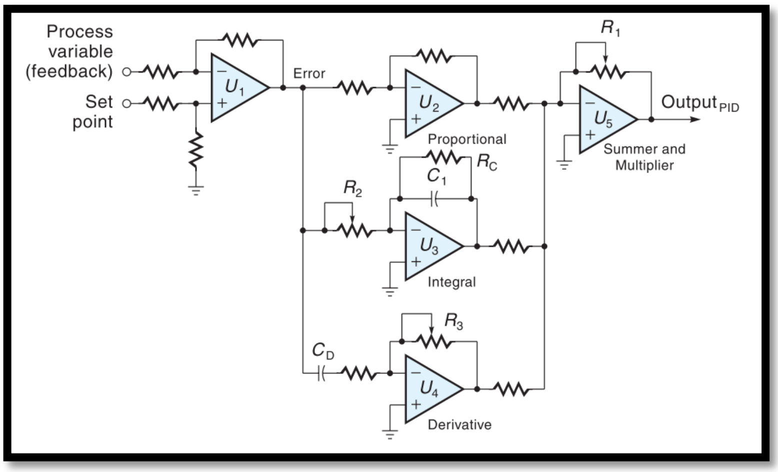

Figure 2.9 An analog PID controller [1]

2.6.2 Digital PID

A PID digital controller is broadly speaking a computer tend likely microprocessor-based. (Feedback Control Principles) state the controller executes a program that performs the same series of operations over and over again. Firstly, the computer key in value of set points (SP) and process variable (PV). Then to solve the system equation 2.4 these data is used and lastly it output the result to the actuator or the actuator drive circuit. The control strategies and parameter can be changed or fined tune by simply modifying the software is the one clear advantages of digital system. Mathematically, the Equation 2.7 below is the PID control system for digital which is can be implemented with a microprocessor-based controller and also knows as difference equation.

$$K_I \sum (E\Delta t) = K_I E_1 T + K_I E_2 T + K_I E_3 T \tag{2.7}$$

2.6.3 PI and PD controller

The PI controller is combination of integrator with a proportional gain. This controller use for first order plant and it will produce a second order response for the system. Generally, if the disturbance or reference inputs either unchanging or have steps change there will be the zero steady-state error.

The Proportional Derivative (PD) is a type of feedback controller whose output, a Control Variable (CV) is generally based on the error (e) between some user-defined Reference Point (RP) and some measured Process Variable (PV)[controller 1]. (Najihah, 2008) state from the PD controller, it can be seen that the derivative controller (K_D) reduces both the overshoot and the settling time. Meanwhile for PI controller, the rise of time is decreased by the integral controller (K_I), both the overshoot and settling time is increased and the steady-state error is eliminated.

The problem for PD controller is to control pendulum's position back to original position which is vertical after some disturbances and therefore the reference signal should be zero. (Hasan, Saha, Rahman, Sarker & Aditya, 2011) said the force applied to the cart was considered as an impulse disturbance. The basic structure of the feedback control system is shown below:

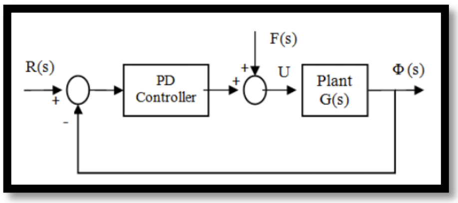

Figure 2.10 Control block diagram (closed loop)

2.6.4 Tuning PID Controller

(Feedback Control Principles) state the method of arriving at numerical values for the constants K_P, K_I and K_D depends on the application. Basically, PID control was applied to process control systems. However nowadays, PID control is being applied to position control systems (such as robots) as well. First, the constants K_P, K_I and K_D are set to initial values, and the controller is connected to the system. The system could consist of the actual hardware or a computer simulation of same. Then the system is operated, and the response is observed. Although many methods of tuning PID controllers exist, two of the most common were developed by Ziegler and Nichols and are called the continuous-cycle method and the reaction-curve method.

The continuous-cycle method (closed-loop method) can be used if the system goes into oscillation. This method will yield a system with a quick response, which means a step-function input will cause a slight overshoot that settles out very quickly. There are 4 steps to tuning PID Controller. First, Set K_P = 1, K_I = 0, and K_D = 0 and connect the controller to the system. Next, using manual control, adjust the system until it is operating in the middle of its range. Then increase the proportional gain (K'_P) while forcing small disturbances to the set point (or the process) until the system oscillates with a constant amplitude. Record the (K'_P) and T_C for this condition. In addition, Based on the values of (K'_P) and T_C from step 2, calculate the initial settings of K_P, K_I* and K_D* as follows:

$$K_P = 0.6K'_P \tag{2.8}$$

$$K_I = \frac{2}{T_C} \tag{2.9}$$

$$K_D = \frac{T_C}{8} \tag{2.10}$$

Using the settings from step 3, operate the system, note the response, and make adjustments as called for. Increasing K_P will produce a stiffer and quicker response, increasing K_I will reduce the time it takes to settle out to zero error, and increasing K_D will decrease overshoot. Of course, K_P, K_I and K_D do not act independently, so changing one constant will have an effect across the board on system response. Tuning the system is an iterative process of making smaller and smaller adjustments until the desired response is achieved.

The reaction-curve method (open-loop method) is another way of determining initial settings of the PID parameters. This method does not require driving the system to oscillation. Instead, the feedback loop is opened, and the controller is manually directed to output a small step function to the actuator. The system response, as reported by the sensor, is used to calculate K_P, K_I and K_D. Note that the actuator, the process itself, and the sensor are operational in this test, so their individual characteristics are accounted for. Because the loop is open, this procedure will work only for systems that are inherently stable. There are 3 steps to tuning using this method. Figure 2.11 shows the steps tuning using the reaction-curve method.

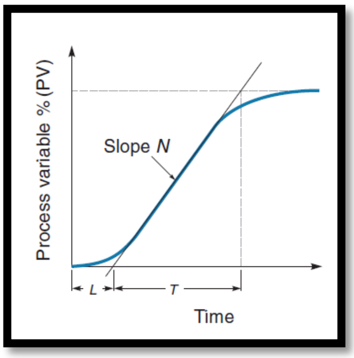

Figure 2.11 Graph of response to step input [1]

Firstly, draw a line tangent to the rising part of the response curve. This line defines the lag time (L) and rise time (T) values. Lag time is the time delay between the controller output and the controlled variable's response. Next, calculate the slope of the curve using the formula below:

$$N = \frac{\Delta PV}{T} \tag{2.11}$$

Where term N is a slope of the system-response curve. ΔPV is change in the process variable, as reported by the sensor (in percentage) and T is a rise time from response curve. Lastly, Calculate the PID constants using formula below:

$$K_P = \frac{1.2\Delta CV}{NL} \tag{2.12}$$

Where ΔCV is a percent step change the in control variable (output of controller), meanwhile N is a slope, as determined by Equation 2.11 and L is a lag time. K_I and K_D are obtained through the formula show in 2.13 and 2.14 below.

$$K_I = \frac{1}{2L} \tag{2.13}$$

$$K_D = 0.5L \tag{2.14}$$

CHAPTER 3

METHODOLOGY

3.1 Introduction

To fulfill requirement for this project, the method should be clear and this project ought to finish within the time. Firstly, the mathematical model of the system is calculated in order to determine the system behavior. These projects are involves hardware and software to completed the project. For the hardware parts, the mechanical part will be designed by using CATIA software and circuit part, driver motor with Arduino UNO used to complete the task. After the design complete, the model and circuit will be build and then software is been used in order to test their function according the plan. Meanwhile for the software, Simulink software had been used to model and simulate the PID controller. The last stage of this project is completed by integrated the hardware and software.

3.2 Mathematical Model

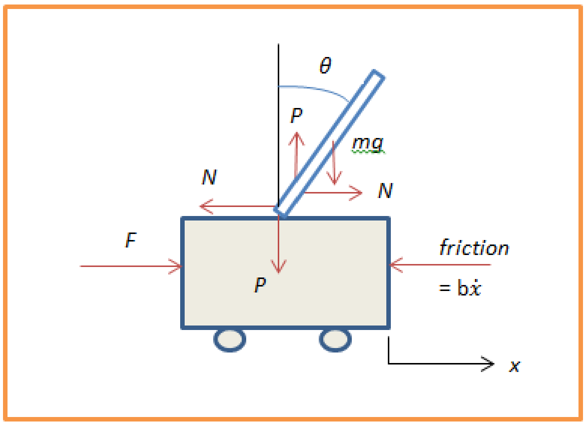

Figure 3.1 Two free-body diagrams of the system.

The mathematical modeling of inverted pendulum is desired by using the newton second law has shown in Equation (3.1) where F is force, m is mass and a for acceleration.

$$F = ma \tag{3.1}$$

The Equation 3.2 is obtained through Equation 3.1, which is in X- axis. The variables M represent the mass of the cart, b is representing the coefficient of friction and N is the force.

$$M\ddot{x} + b\dot{x} + N = F \tag{3.2}$$

In order to get value N, the Equation 3.3 is used. The variables m represent the mass of pendulum, and l is representing the half of length pendulum.

$$N = m\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}\sin\theta \tag{3.3}$$

Next, the Equation 3.2 and 3.3 is combined in order to summarize the equation.

$$(M + m)\ddot{x} + b\dot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = F \tag{3.4}$$

The equation through Y – axis is obtained and summarize in one equation which is shown on Equation 3.5 below.

$$P\ \sin\theta + N\ \cos\theta - mg\sin\theta = ml\ddot{\theta} + \ m\ddot{x}\ \cos\theta \tag{3.5}$$